VECTOR ALGEBRA

Assignment 4 Practice *by* O.P. GUPTA • M. +91-9650350480

Q01. If $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+p\hat{j}+q\hat{k})=\vec{0}$, then the value of p and q are

(a) $p=6,\ q=27$ (b) $p=3,\ q=\frac{27}{2}$ (c) $p=6,\ q=\frac{27}{2}$ (d) $p=3,\ q=27$

Q02. The magnitude of projection of $(2\hat{i}-\hat{j}+\hat{k})$ on $(\hat{i}-2\hat{j}+2\hat{k})$ is

(a) 2 (b) 3 (c) 1 (d) 0

Q03. Vector of magnitude 5 units and in the direction opposite to $2\hat{i}+3\hat{j}-6\hat{k}$ is

(a) $\frac{5}{7}(-2\hat{i}-3\hat{j}-6\hat{k})$ (b) $\frac{5}{7}(-2\hat{i}-3\hat{j}+6\hat{k})$ (c) $(-2\hat{i}-3\hat{j}+6\hat{k})$ (d) $\frac{1}{7}(-2\hat{i}-3\hat{j}+6\hat{k})$

Q04. A unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$ is

(a) $\hat{j}$ (b) $\hat{i}$ (c) $\hat{k}$ (d) $\frac{3}{4}\hat{j}$

Q05. Area (in Sq. units) of the triangle whose two sides are represented by the vectors $2\hat{i}$ and $-3\hat{j}$, is

(a) 1 (b) 2 (c) 3 (d) 4 3

Q06. Angle between the unit vectors $\hat{a}$ and $\hat{b}$, given that $|\hat{a}+\hat{b}|=1$, is

(a) $\frac{5\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Q07. If $\hat{i},\ \hat{j},\ \hat{k}$ are unit vectors along three mutually perpendicular directions, then

(a) $\hat{i}.\hat{j}=1$ (b) $\hat{i}\times\hat{j}=1$ (c) $\hat{i}.\hat{k}=0$ (d) $\hat{i}\times\hat{k}=0$

Q08. ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$ equals

(a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{AD}$

Q09. If $\vec{a}.\vec{b}=\frac{1}{2}|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) 0° (b) 30° (c) 60° (d) 90°

Q10. If $\vec{a}$ is a non-zero vector, then $(\vec{a}.\hat{i})\hat{i}+(\vec{a}.\hat{j})\hat{j}+(\vec{a}.\hat{k})\hat{k}$ equals

(a) $\vec{0}$ (b) $-\vec{a}$ (c) $\vec{a}$ (d) $2\vec{a}$

Q11. The projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$ is

(a) $\vec{0}$ (b) 1 (c) 2 (d) 0

Q12. Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a}.\vec{b}=|\vec{b}|^2$ and $|\vec{a}-\vec{b}|=\sqrt{7}$, then $|\vec{b}|$ equals

(a) 7 (b) 14 (c) $\sqrt{7}$ (d) 21

Q13. The value of p for which $p(\hat{i}+\hat{j}+\hat{k})$ is a unit vector is

(a) 0 (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\sqrt{3}$

Q14. The area of the parallelogram, whose diagonals are $2\hat{i}$ and $-3\hat{k}$, is _______ square units.

(a) 2 (b) 3 (c) 1 (d) $\sqrt{3}$

Q15. The value of λ for which the vectors $2\hat{i}-\lambda\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-\hat{k}$ are orthogonal, is

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

Q16. If $|\vec{a}|=4$ and $-3\le\lambda\le 2$ then $|\lambda\vec{a}|$ lies in

(a) [0,12] (b) [2,3] (c) [8,12] (d) [−12,8]

Q17. The area of a triangle formed by vertices O, A and B, where $\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k}$ and $\overrightarrow{OB}=-3\hat{i}-2\hat{j}+\hat{k}$ is

(a) $3\sqrt{5}$ sq. units (b) $5\sqrt{5}$ sq. units (c) $6\sqrt{5}$ sq. units (d) 4 sq. units

Q18. The angle between the vectors $\hat{i}-\hat{j}$ and $\hat{j}-\hat{k}$ is

(a) $-\frac{\pi}{3}$ (b) 0 (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Q19. If $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{a}\times\vec{b}|=6$, then the value of $\vec{a}.\vec{b}$ is

(a) 12 (b) 6 (c) $3\sqrt{3}$ (d) $6\sqrt{3}$

Q20. If the projection of $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ on $\vec{b}=2\hat{i}+\lambda\hat{k}$ is zero, then the value of λ is

(a) 0 (b) 1 (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

Q21. The position vectors of two points A and B are $\overrightarrow{OA}=2\hat{i}-\hat{j}-\hat{k}$ and $\overrightarrow{OB}=2\hat{i}-\hat{j}+2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio $2:1$ is

(a) $2\hat{i}-\hat{j}-\hat{k}$ (b) $2\hat{i}+\hat{j}-\hat{k}$ (c) $2\hat{i}-\hat{j}+\hat{k}$ (d) $\hat{i}-2\hat{j}+\hat{k}$

Q22. If $\vec{a}=\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$, then the value of λ is

(a) 1 (b) −1 (c) 2 (d) −2

Q23. Write the projection of vector $\vec{r}=3\hat{i}-4\hat{j}+12\hat{k}$ on (i) x-axis, and (ii) y-axis.

Q24. If $\vec{a}=\alpha\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{b}=2\hat{i}-\hat{j}-\beta\hat{k}$, then find the value of α and β so that $\vec{a}$ and $\vec{b}$ may be collinear.

Q25. Find the magnitude of vector $\vec{a}$ given by $\vec{a}=(\hat{i}+3\hat{j}-2\hat{k})\times(-\hat{i}+3\hat{k})$.

Q26. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, find the value of $|\vec{a}+2\vec{b}+3\vec{c}|$.

Q27. If the sides AB and BC of a parallelogram ABCD are represented as vectors $\overrightarrow{AB}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\overrightarrow{BC}=\hat{i}+2\hat{j}+3\hat{k}$, then find the unit vector along diagonal AC.

Q28. If $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=5\hat{i}-3\hat{j}-4\hat{k}$, then find the ratio $\dfrac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$.

Q29. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

Q30. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{i}-\hat{j}+\hat{k}$ and $4\hat{i}+5\hat{k}$ respectively.

Q31. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}=5\hat{i}+6\hat{j}-2\hat{k}$ and $\vec{b}=7\hat{i}+6\hat{j}+2\hat{k}$.

Q32. Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

Q33. Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\,\vec{a}-\vec{b}$ is also a unit vector.

Q34. Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$,

$|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

Q35. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $3\hat{i}+7\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+2\hat{k}$ form the sides of a right-angled triangle.

Q36. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$ then, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

Q37. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

Q38. Using vectors, find the area of the triangle with vertices A(1, 2, 3), B(2, –1, 4) and C(4, 5, –1).

Q39. Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}|=2|\vec{b}|$ and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=12$.

Q40. If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove that $\sin\frac{\theta}{2}=\frac{1}{2}|\hat{a}-\hat{b}|$.

Q41. Show that $|\vec{a}|\vec{b}+|\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b}-|\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.